ANNULAR CAPILLARY WAVES

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A system of annular capillary waves which forms with a droplet falling on a liquid surface is studied. Interest in this is governed in particular by the fact that the annular capillary waves which form affect the result of impact of a droplet with a surface which occurs in multiphase flows used in chemical technology. From a theoretical point of view annular capillary waves caused by a falling droplet are related to the family of waves formed by a single pulse on a liquid surface. Previously [1-3] annular waves have been considered which are formed by a point source for the case of gravity waves and gravity-capillary waves caused by a rain droplet [4]. Experimental studies of the family of waves mentioned above are fragmentary [5] and there is no information about fundamental capillary waves.

In the present work a study is made of the development of a system of annular capillary waves which form with a droplet of radius $R = (0.8\text{-}2.0)\cdot 10^{-4}$ m falling on a liquid surface. The results obtained are compared with the theory.

We consider capillary waves which occur at the surface of a liquid when a droplet of radius R falls on it. We use a cylindrical coordinate system (r, z, θ) with a center at the point of impact. Axis z is directed upwards from the water level. By assuming the liquid to be incompressible and ignoring dissipation processes the wave movement is assumed to be potential. The velocity potential $\phi(r, z, t)$ satisfies the Laplace equation within the liquid $\Delta\phi = 0$. Boundary conditions at the free surface of the liquid η are as follows:

$$\partial\eta/\partial t = \partial\varphi/\partial n,\ \partial\varphi/\partial t = \sigma/\rho(1/R_1 + 1/R_2)$$

(R_1, R_2 are principal radii of curvature for the surface, and σ, ρ are surface tension coefficient and liquid density).

In prescribing the initial conditions we assume that at instant of time t = 0 a disturbance is represented by an initial pulse of the surface in the region of impact:

$$\varphi(r, 0) = -I_0(r) = \begin{cases} I, & 0 \leqslant r \leqslant R, \\ 0, & r < R. \end{cases}$$

The magnitude of the initial pulse is connected with the pulse of the droplet.

The current problem is similar to that considered in [4] for gravity-capillary waves. Repeating the analysis provided in [4] but only taking account of capillary effects it is possible to obtain a solution for annular capillary waves in the form

$$\eta(r, t) = -\frac{1}{4\pi} I \frac{\sigma}{\rho} \frac{R^3 k}{r} \sqrt{\frac{W'(k)}{|W''(k)|}} \sin(wt - kr), \tag{1}$$

which is correct with the conditions $kr \gg 1$, $kR < 1$. Here $W(k) = (\sigma k^3/\rho)^{1/2}$ is the root of the dispersion relationship for capillary waves: $W'(k)$, $W''(k)$ are first and second derivatives of $W(k)$; $w = \pm W(k)$ is frequency; k is wave number.

It can be seen from (1) that the size and shape of the initial disturbance to a first approximation only affects the amplitude of oscillations, but it does not affect the nature of change with time of the wave picture. By using dependences for capillary waves [1]

$$k = \frac{4r^2\rho}{9\sigma t^2},\quad w = \frac{8}{27}\frac{r^3\rho}{\sigma t^2},$$

we rewrite (1) as

$$\eta(r, t) = \frac{8R^3}{27\sqrt{2\pi}} I \frac{r^3\rho}{\sigma t^4} \sin\left(\frac{4}{27}\frac{r^3\rho}{\sigma t^2}\right).$$

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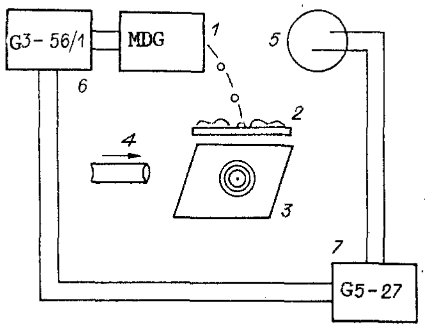

Fig. 1

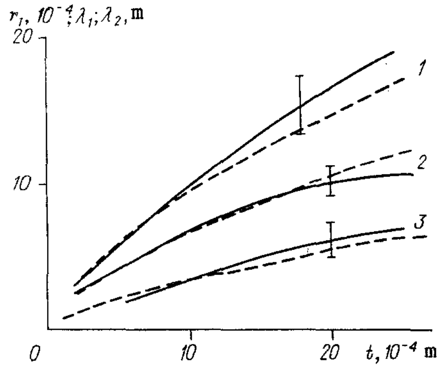

Fig. 2

It is understood that the maximum values of $\eta(r, t)$ are governed by the maximum values of function $\sin\left(\frac{4}{27}\frac{r^3\rho}{\sigma t^2}\right)$. Proceeding from this it is possible to write a rule for the change with time of the coordinates of the maximum elevation of the free surface, i.e., the coordinates of the crests:

$$r_m = \left\{(\pi/2 + 2\pi m)\frac{27\sigma}{4\rho}\right\}^{1/3} t^{2/3} \qquad (2)$$

(m = 0, 1, 2,... for the first and subsequent crests respectively).

Experimental observation of a system of annular capillary waves which form with a droplet falling on a water surface was carried out by means of a procedure for visualizing rapid processes [6]. A block diagram of the device is presented in Fig. 1. Water droplets formed by a monodispersed droplet generator (MDG) 1 of the 'vibrating needle' type [7] fall into a vessel with a transparent bottom filled with water 2. The error in MDG operation is not more than 1%. Ranges for the falling droplet radius R and velocity v were varied within the limits $R = (0.8 - 2.0)\ 10^{-4}$ m, $v = 0.8$-2.0 m/sec. The impact frequency was selected as 40-60 Hz, sufficiently low so that in the time between impacts (10^{-2} sec) the liquid surface manages to recover its properties. According to estimates in [8] the relaxation time for the water surface is 10^{-3} sec.

Thus the accuracy of the wave picture formed by individual impacts repeated that of the previous one. A mirror 3 was placed beneath the vessel at an angle of 45° to the surface of the liquid and the focal plane of microscope 4 through which the process was observed. The vessel surface was lit with sources of constant and pulsed illumination 5. The optical system was tuned so that a sharp image of rings was obtained in the mirror corresponding the maximum elevations of the wave surface.

The master MDG generator G3-56/1 6 was synchronized with the triggering generator of the pulsed lamp through a pulse delay unit G5-27 7 which made it possible to delay the pulse in the range 10^{-6}-10^{-1} sec. By fixing the instant of droplet impact with the surface and making a time shift by a known amount with a stroboscopic picture the radius of a ring r_m was determined at different instants of time. The measurement accuracy was limited by the size of the microscope scale divisions. The absolute measurement error for r_m: $\Delta r_m = 0.25 \times 10^{-4}$ m. Experiments were performed with distilled water at 20°C.

Observation of the development of a wave system formed by droplets with different parameters confirmed the theoretical concept that the radius of a falling droplet R and its velocity v do not affect the rule for the change of the wave picture with time.

From an initial localized disturbance at the point of impact a group of annular capillary waves diverge consisting of three crests. The crest velocity and distance between them increases constantly. A splash develops at the point of impact. Breakdown of the splash leads to development of a second system of annular waves diverging from the center. The length of waves in it is much shorter than the waves of the first system. This wave system attenuates without managing to reach the adjacent crest of the main system and it was not specially studied.

The instant when the droplet touched the surface was recorded in each experiment. Then from a certain instant of time when $r_1 > R$ (r_1 is distance from the center to the first crest) through a time interval of $\Delta t = 10^{-4}$ sec distance r, and the distance between crests were determined: $\lambda_1 = |r_1 - r_2|$, $\lambda_2 = |r_2 - r_3|$.

Represented in by curves 1-3 in Fig. 2 are the time dependences obtained $r_1(t)$, $\lambda_1(t)$, and $\lambda_2(t)$. Each curve is a summary of experimental data for a wave system formed by droplets with different parameters. Time t = 0 is the instant of droplet contact with the surface. Broken lines refer to theoretical dependences which are easily obtained from (2) assuming that $\sigma = 0.72$ N/m, $\rho = 1000$ kg/m^3, m = 0, 1, 2 for r_1, r_2, r_3 respectively: $r_1 = 8.9\ t^{2/3}$, $\lambda_1 = r_1 - r_2 = 6.3\,t^{2/3}$, $\lambda_2 = r_2 - r_3 = 3.2\ t^{2/3}$. The good agreement of experimental and theoretical data can be seen from Fig. 2.

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INFLUENCE OF CAPILLARY FORCES ON THE NON-STATIONARY FALL OF A DROP IN AN UNBOUNDED FLUID

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A large number of works are devoted to the problem of the dynamics of a viscous fluid drop (see [1-13]). At present the problem of the motion of a drop under the action of surface tension forces is of particular interest. These forces depend in essence on the temperature and concentration of surface-active substances (SAS) at the boundary separating the fluids. This interest is primarily determined by the requirements of chemical technology [3, 7] and the development of space studies [5, 14], where one must be able to predict the behavior of fluids in weak force fields and in conditions of weightlessness. In this work, an explicit solution is constructed for the linear problem of drop motion in an unbounded fluid with SAS present. The number of SAS is arbitrary (to a first approximation, chemical reactions are not taken into account).

Let a drop of viscous, incompressible fluid be located in another fluid with low SAS concentrations in solution, and let the drop begin slow motion under the action of variable acceleration of a volume force $\mathbf{g}(t)$ (t is the time). As a result of dilatation of the separation boundary Γ, the thermodynamic equilibrium of SAS is displaced in the volume and at the surface. This leads to an additional capillary force, which retards the motion of the drop. In other words, the Le Chatelier principle holds: the external action on a system in a stable state of thermodynamic equilibrium produces a reaction in the system which reduces the effect of the external action. The latter important property of reactivity of capillary forces is directly related to the fundamental principles of thermodynamics. Consistent application of these principles makes it possible to write the equations of thermal

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